Approximate Streaming Regular Pattern Matching

Adrien Mathieu

Thursday 29th, February

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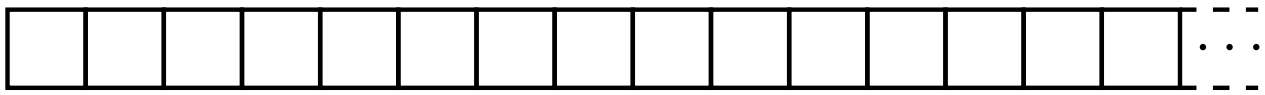

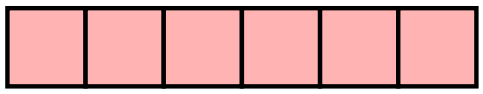

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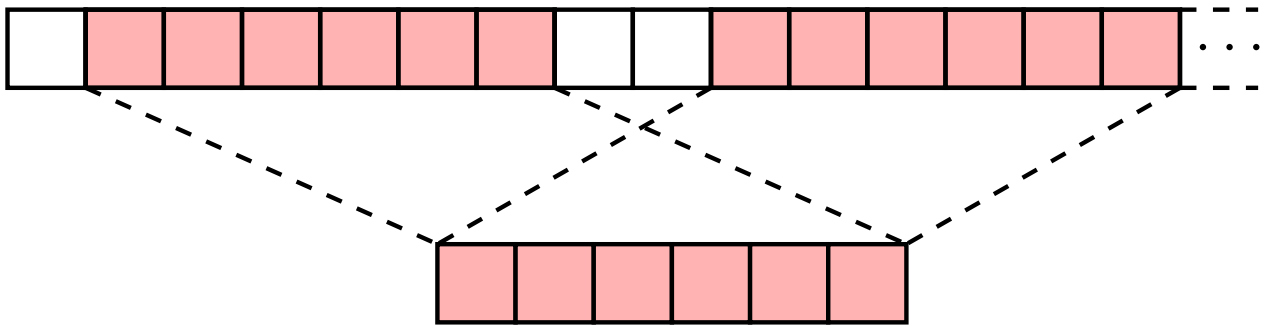

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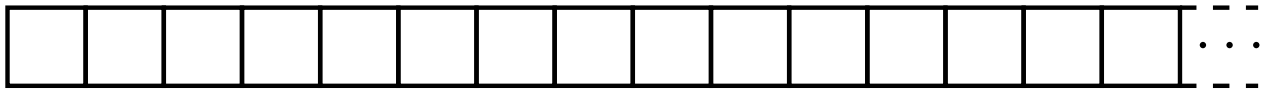

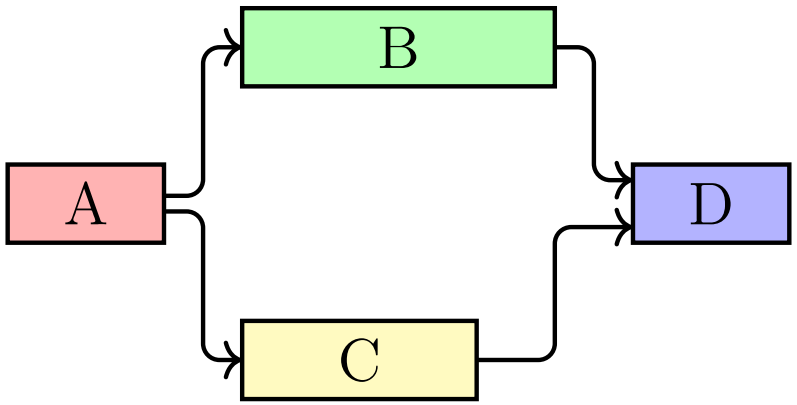

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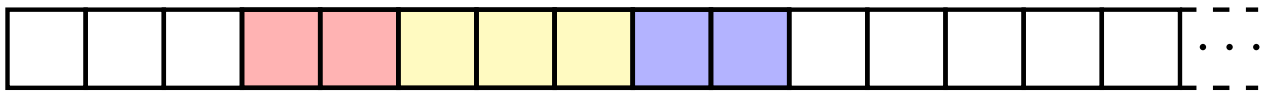

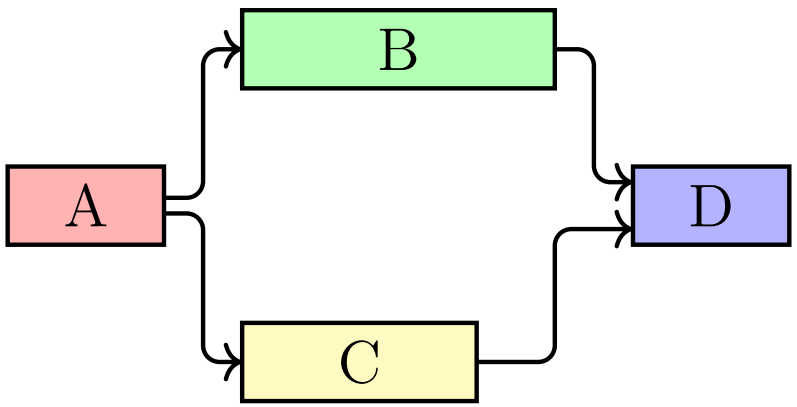

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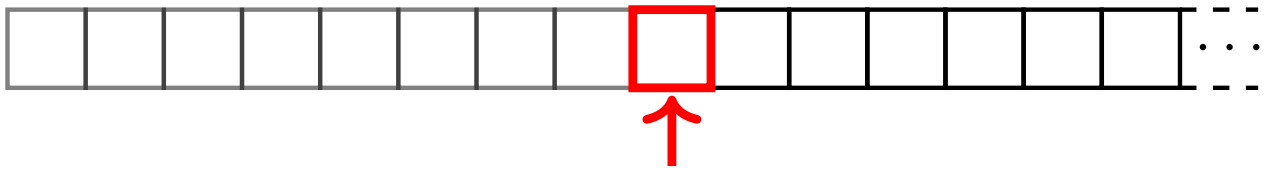

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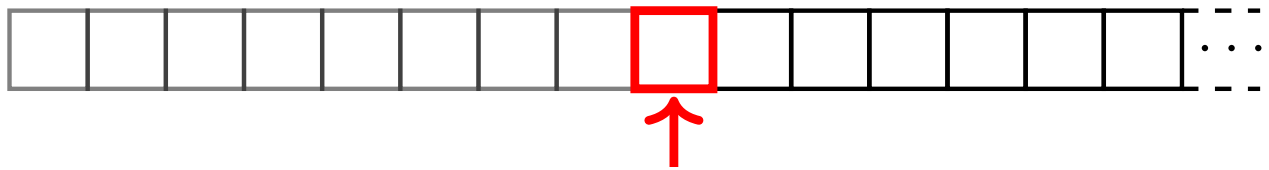

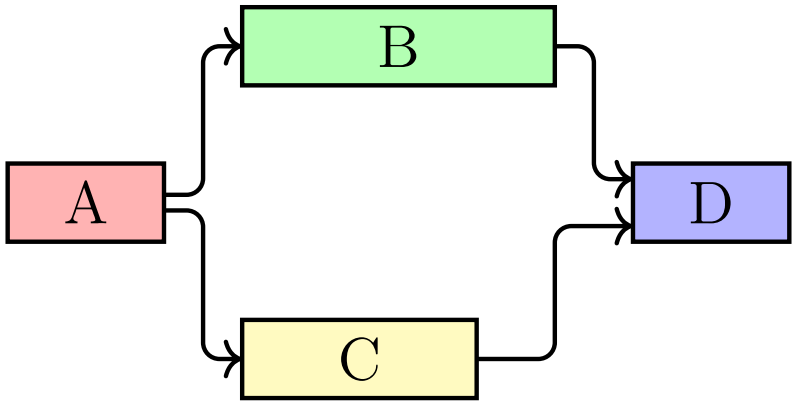

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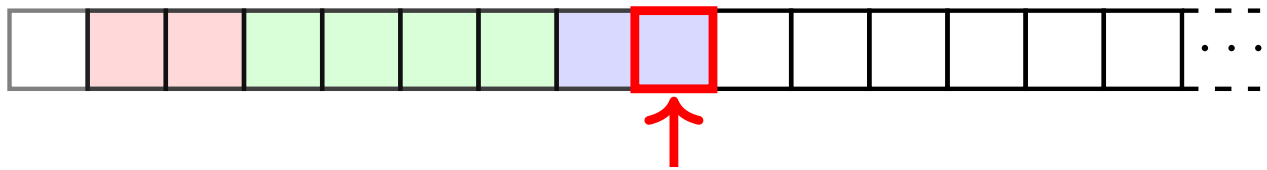

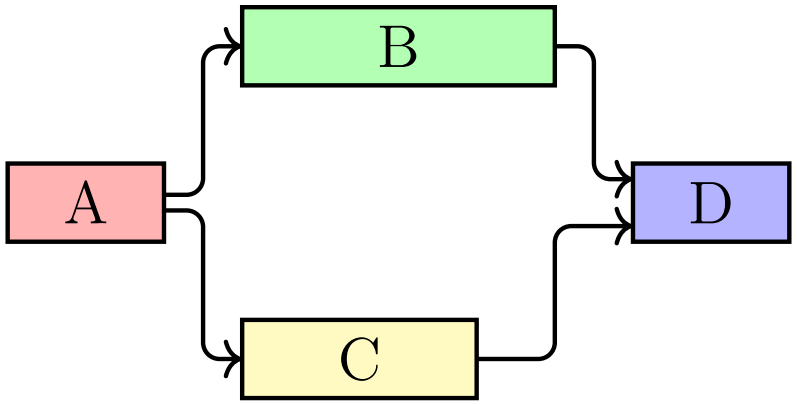

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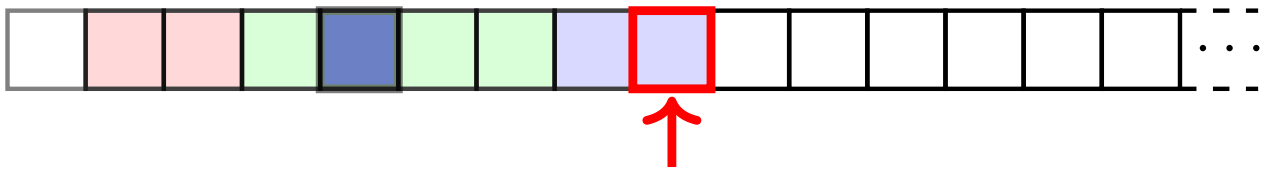

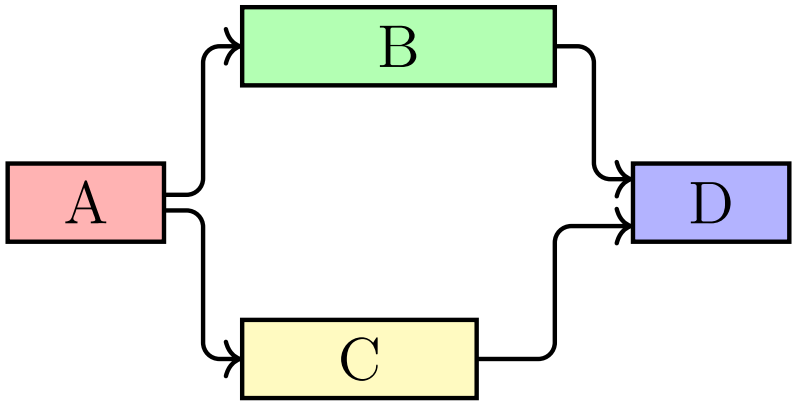

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Where are we going?

1. Exact Regular Expression Matching
   - Solution Sketch
   - Witnesses
   - Space-Time Tradeoff
   - Backtracking Solution
   - Occurrence Structure
   - Precomputing Backtracking Queries
   - Reducing to a Graph Problem
   - Reducing to a Circuit Problem
2. Generalizing to Approximate Matching
   - General Idea
   - Handling Arithmetic Progressions
   - The Stubborn Edge Case

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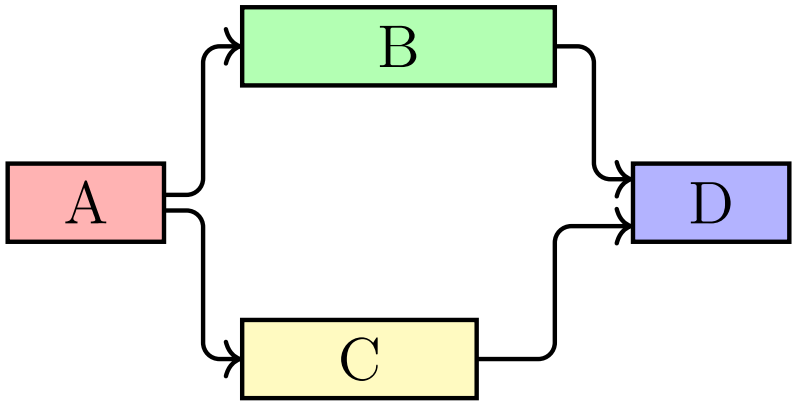

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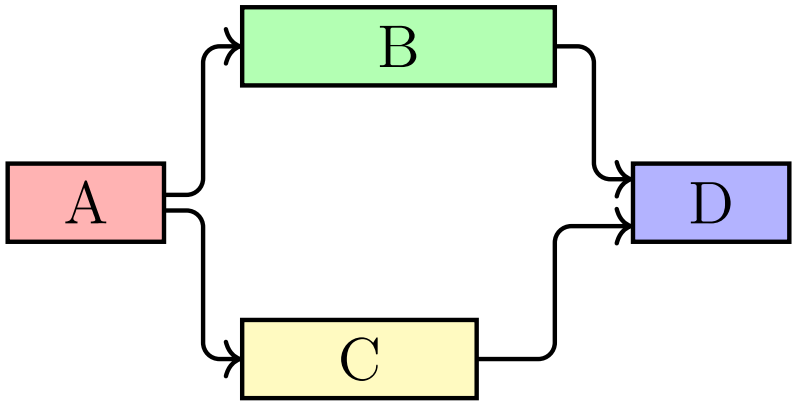

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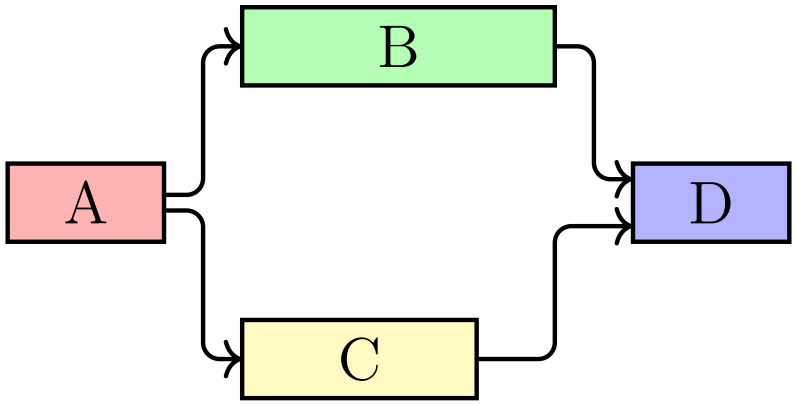

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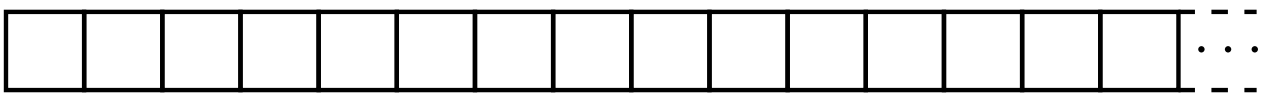

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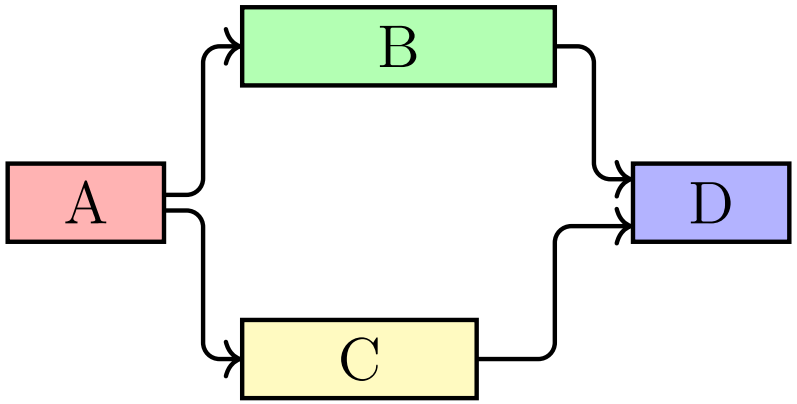

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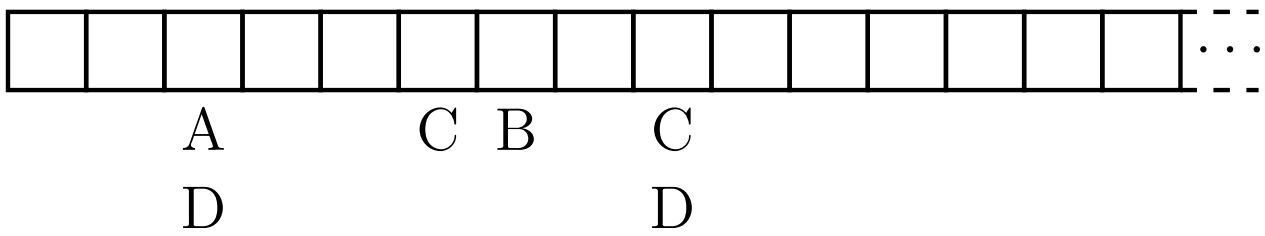

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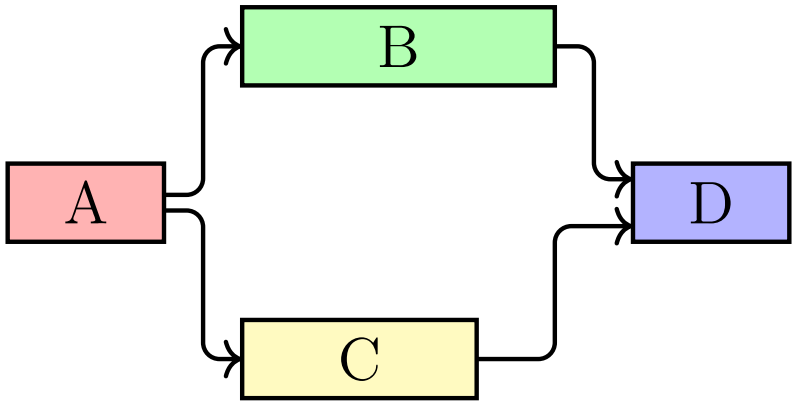

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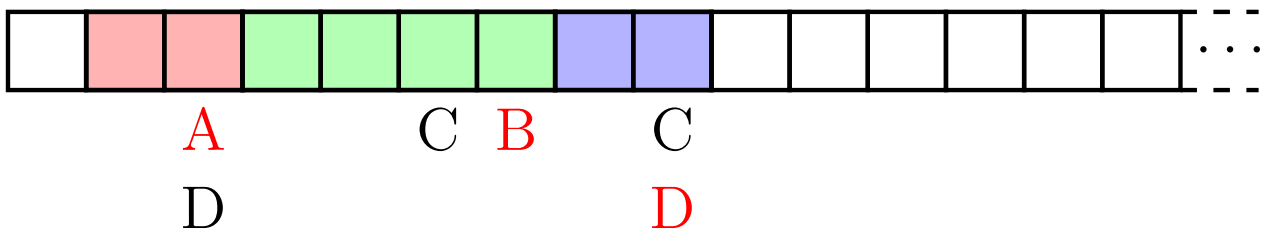

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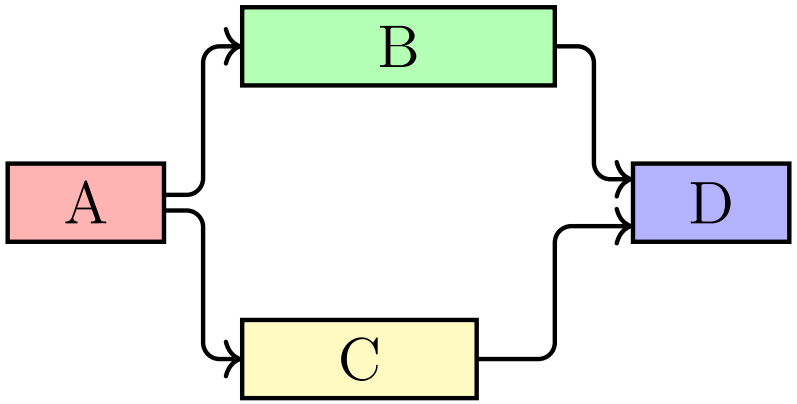

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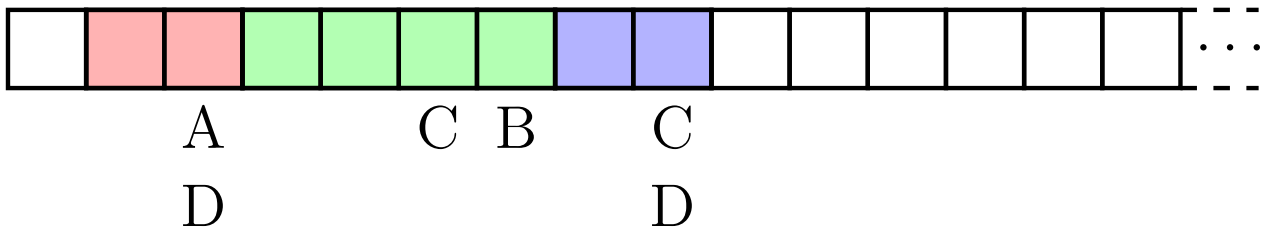

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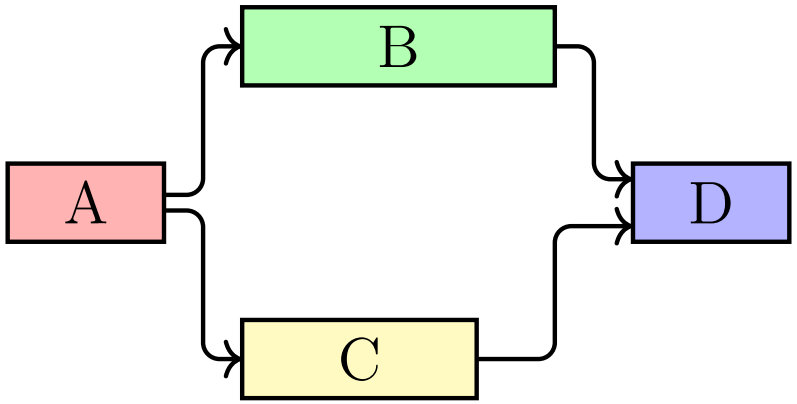

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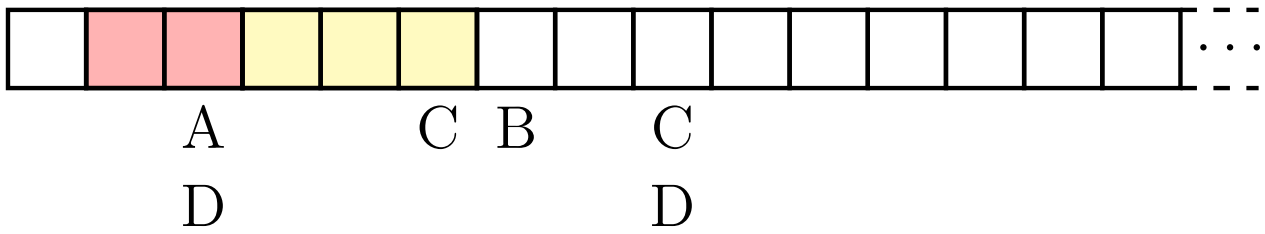

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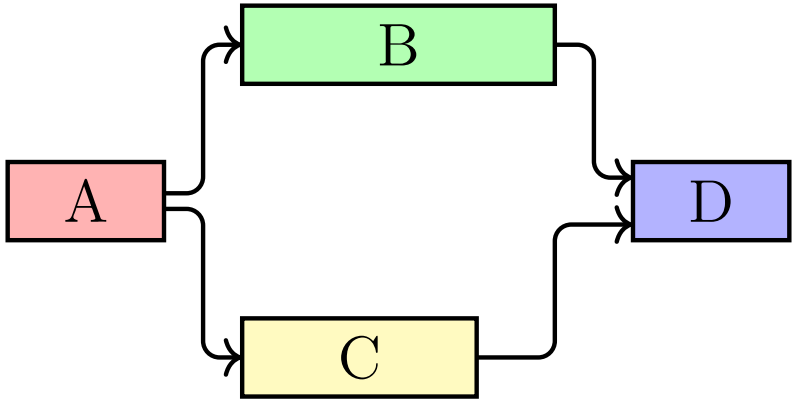

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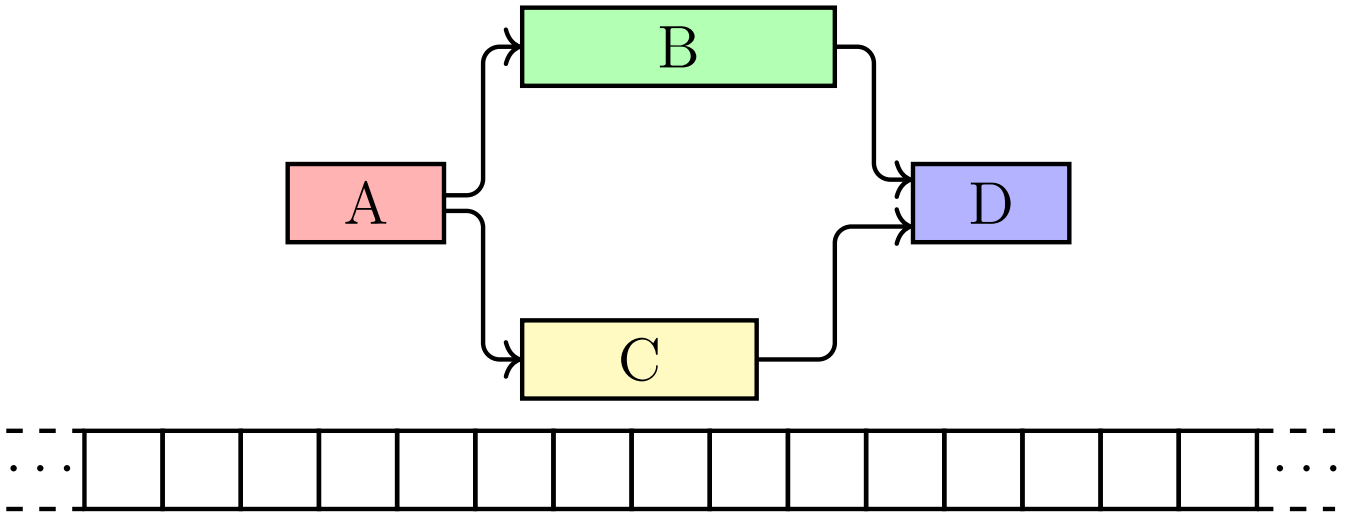

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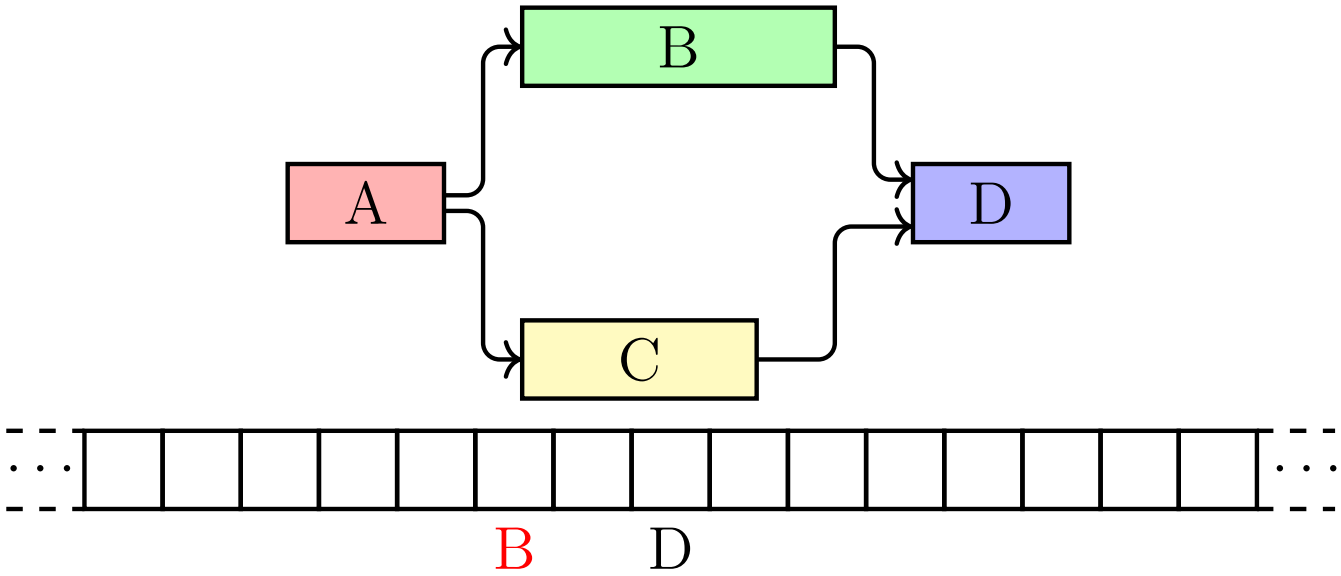

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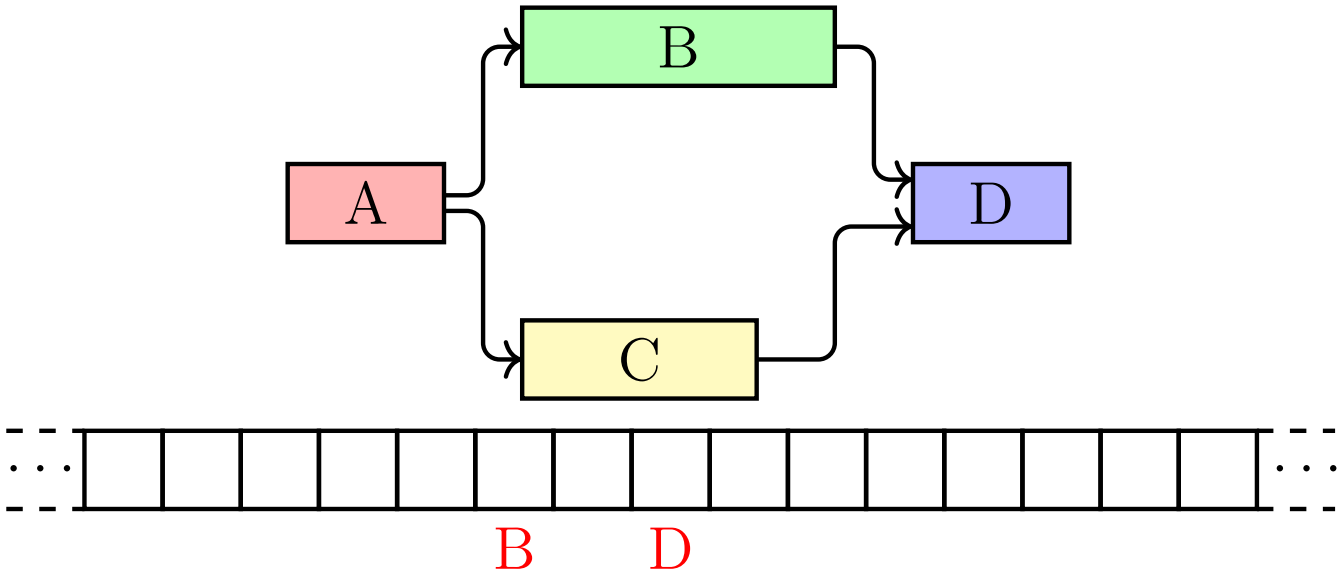

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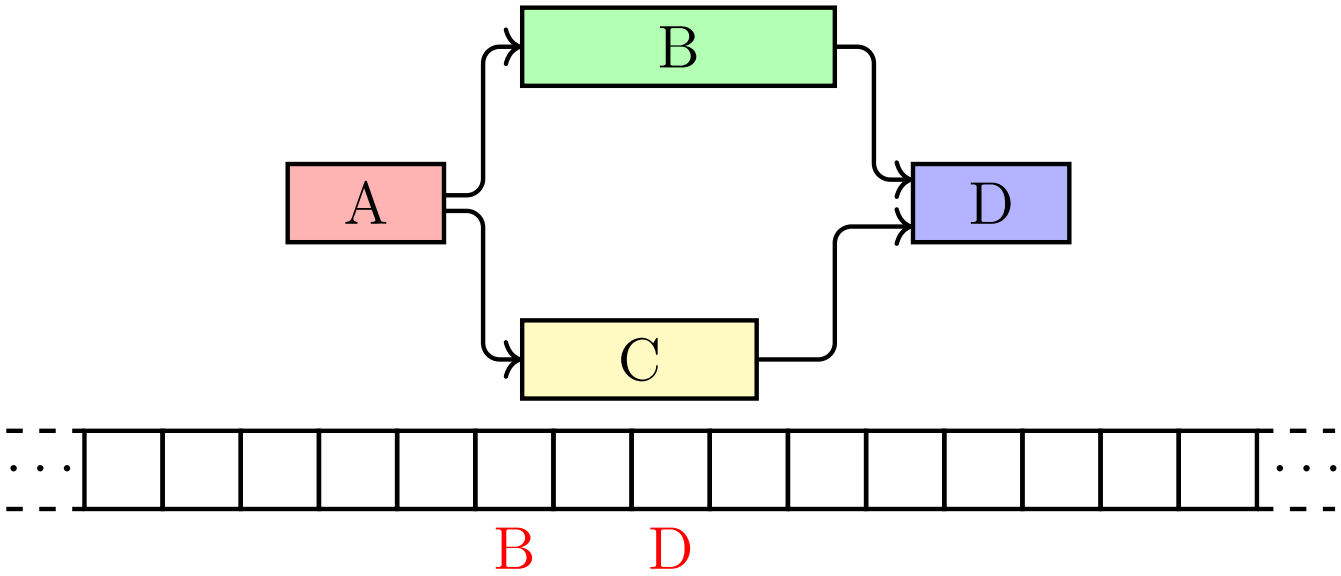

Strategy

- remember all witnesses
- for each character read, find all atomic strings for which it's an occurrence
- for each such occurrences, decide whether they are witnesses

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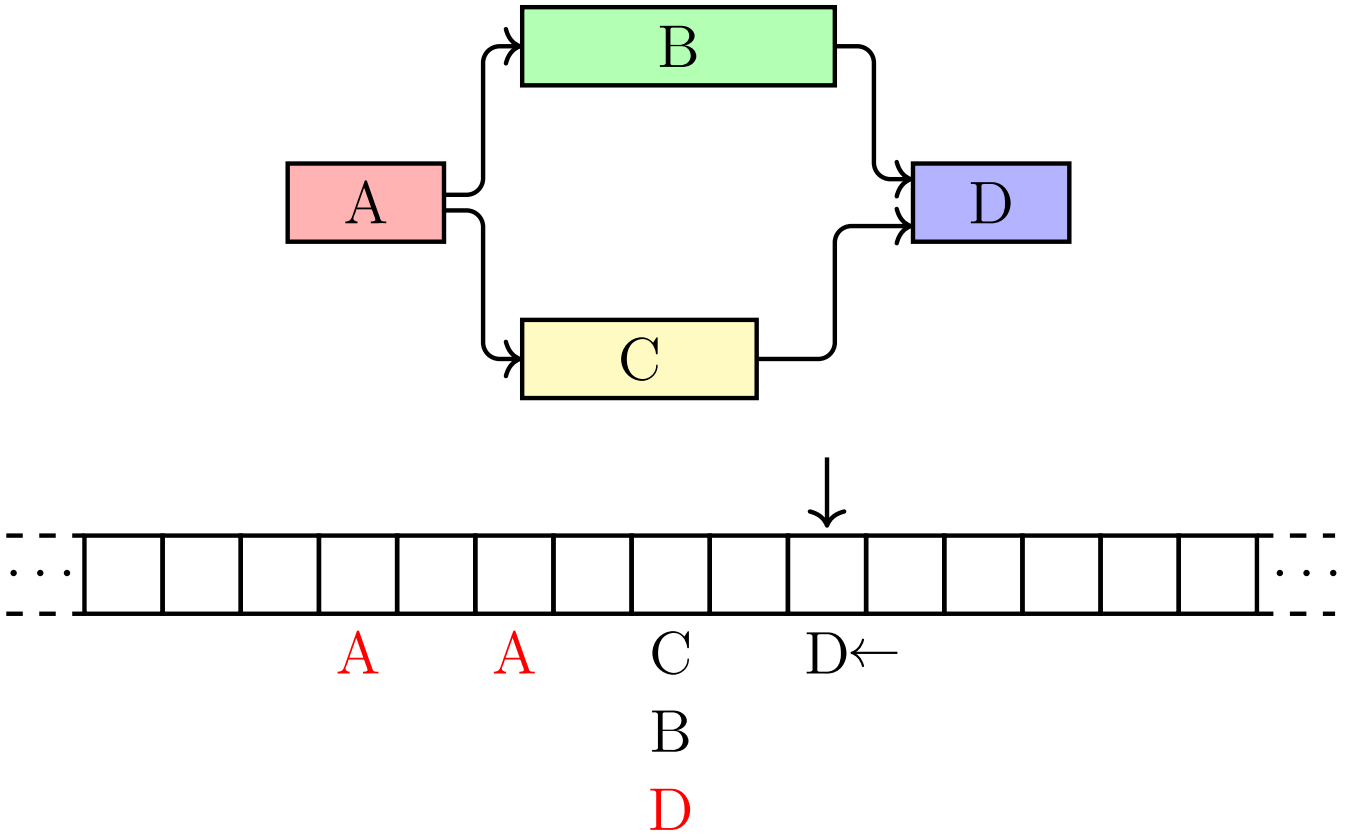
This is still enough information to decide whether a given occurrence is a witness.

Backtracking Solution

To find whether a given occurrence is a witness, we consider occurrences of preceding string in the automaton.



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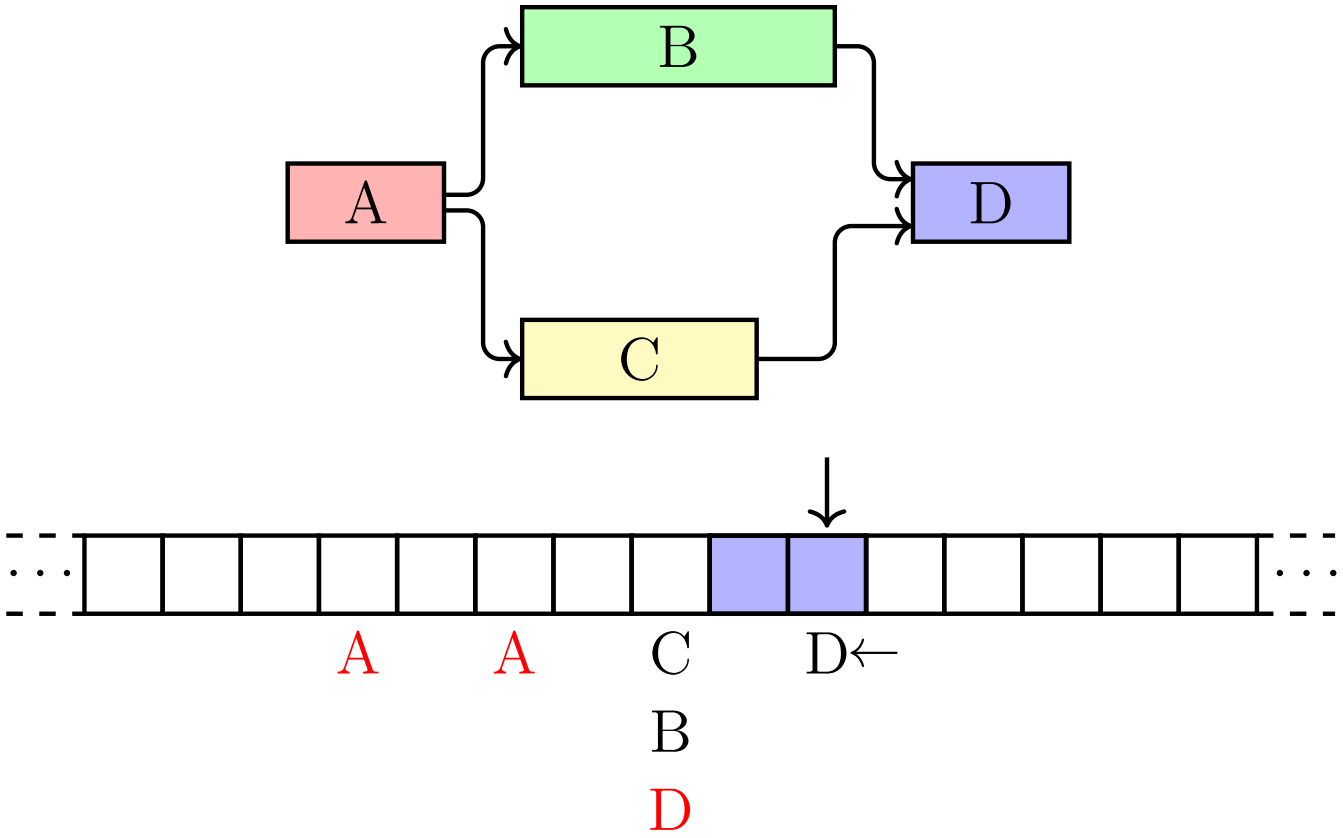

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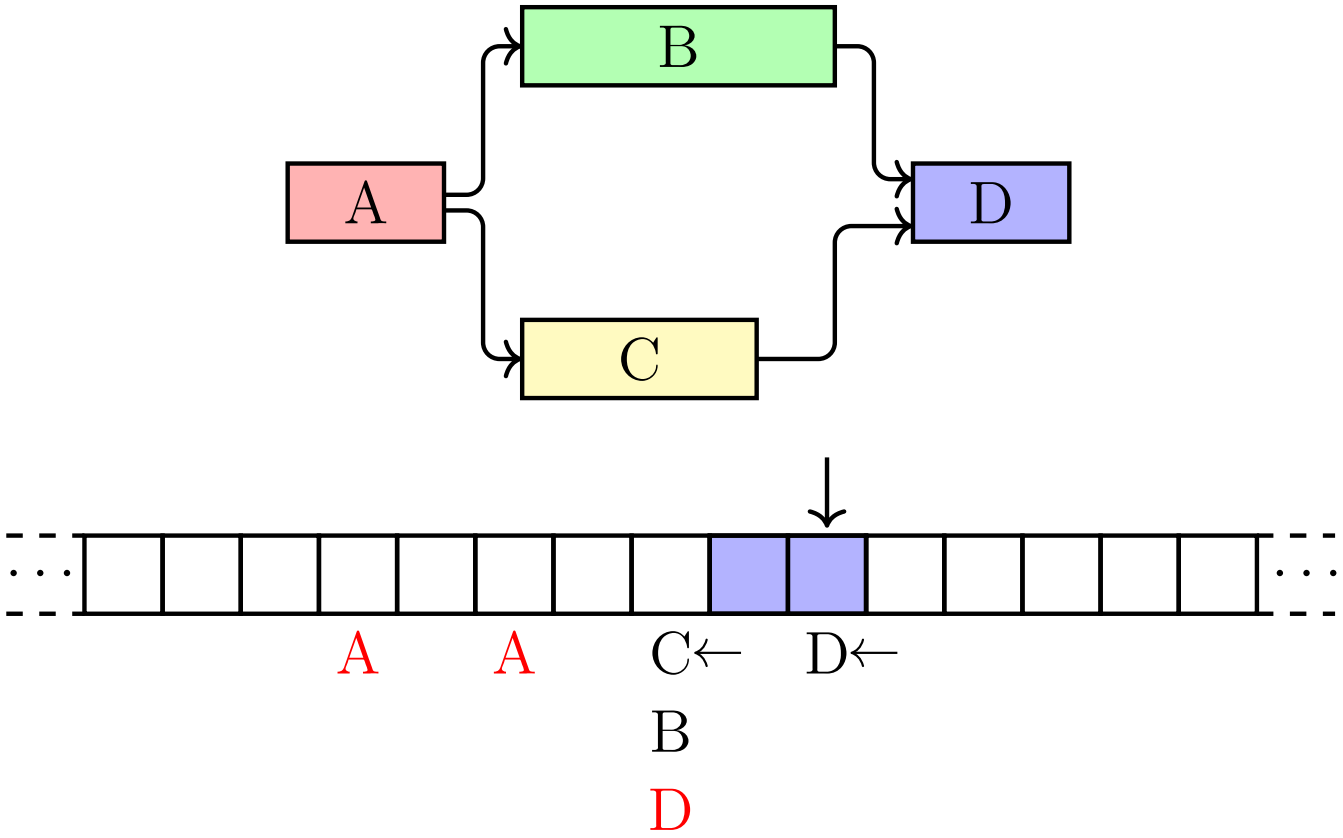

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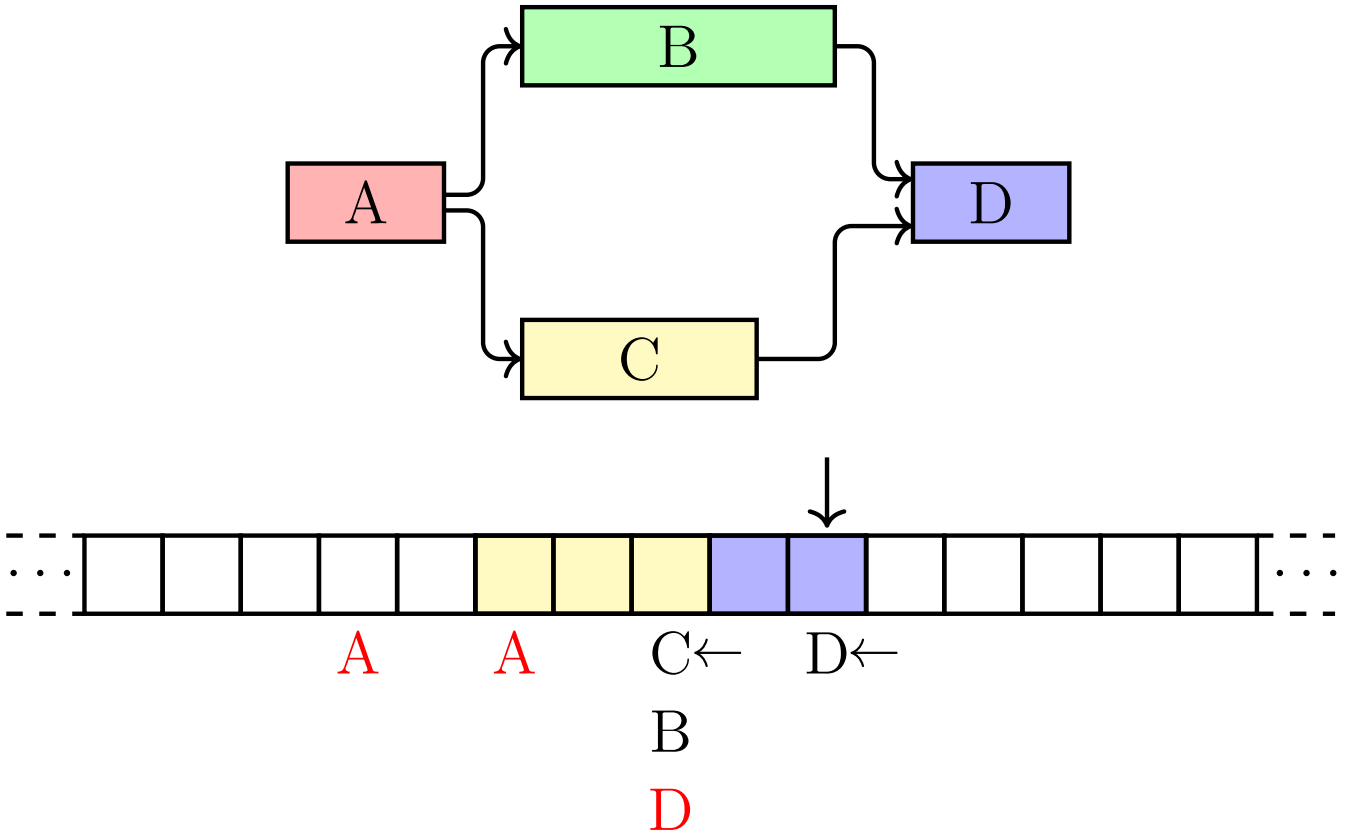

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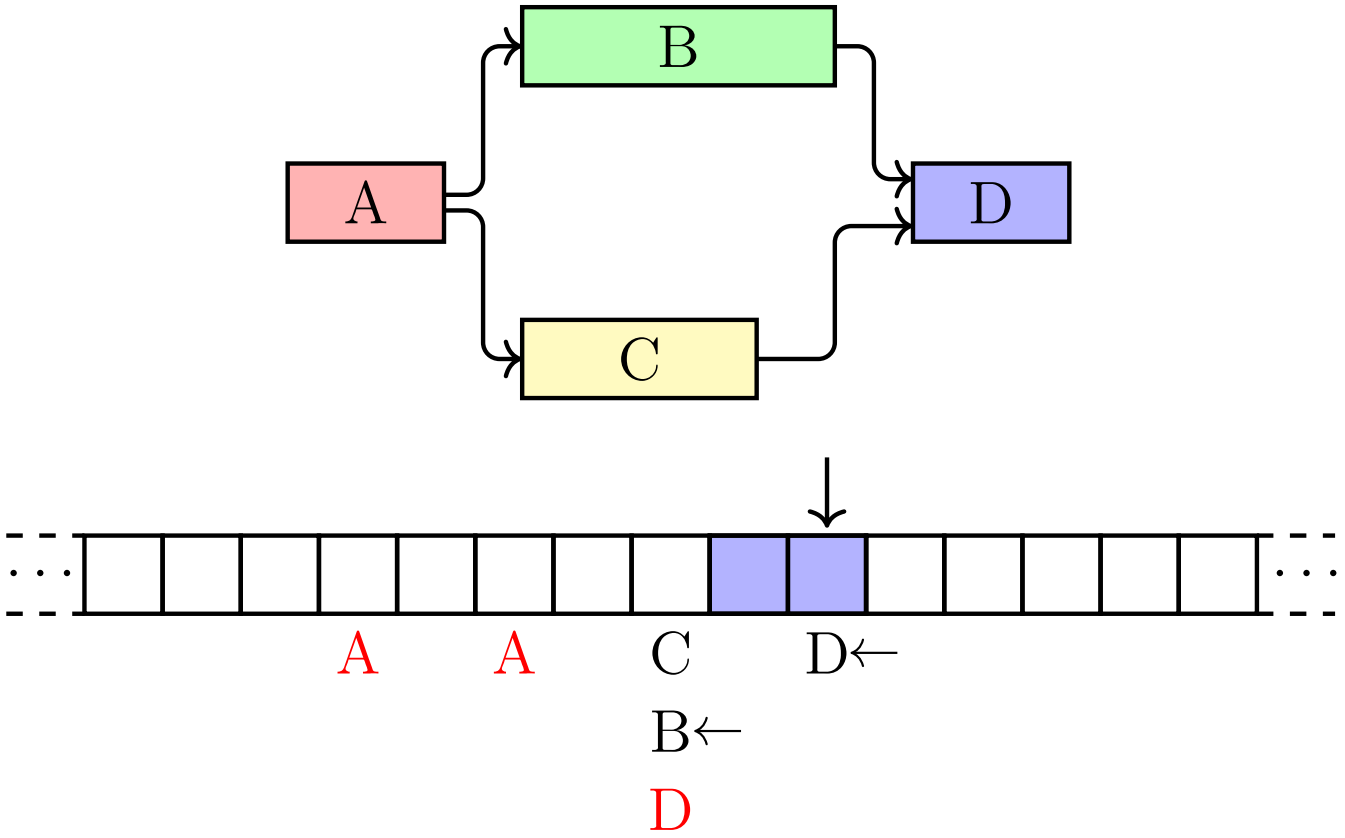

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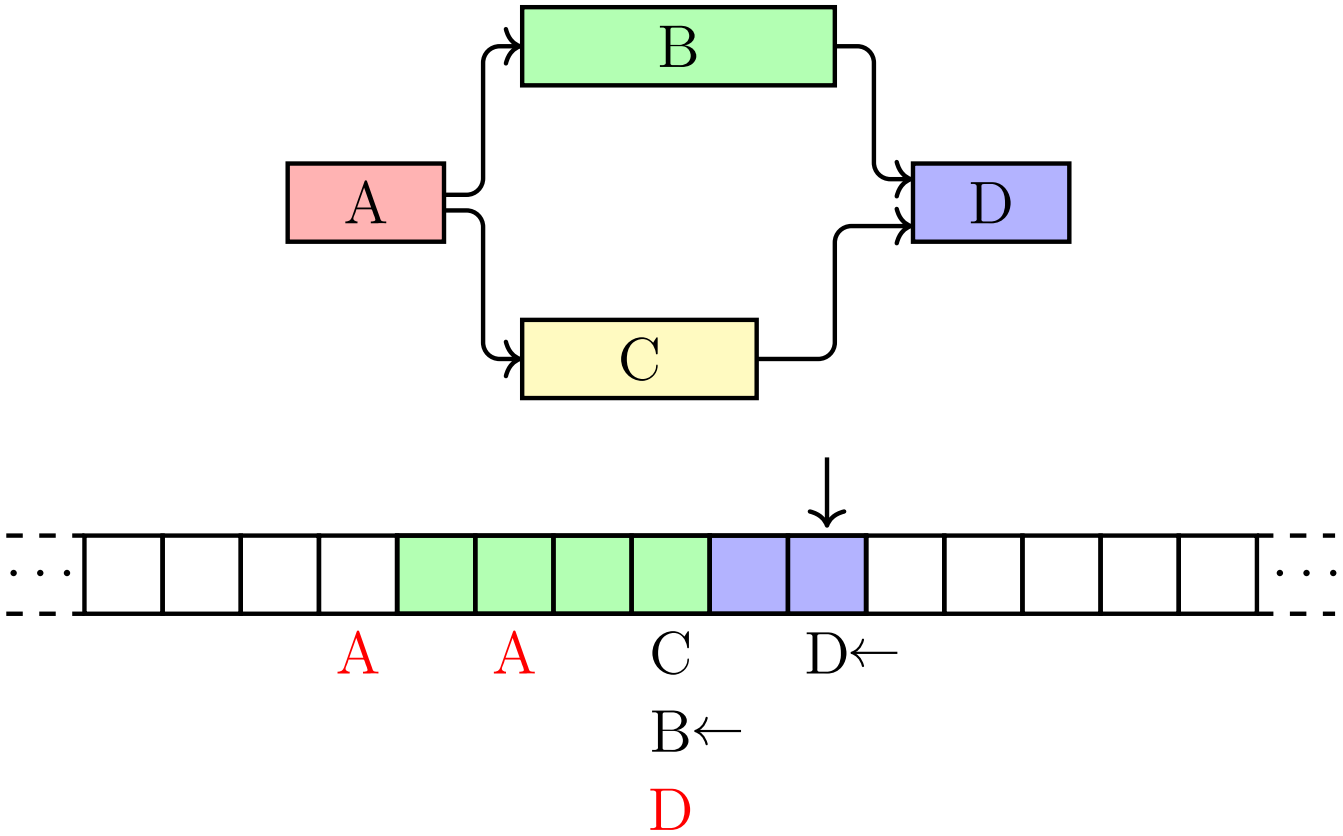

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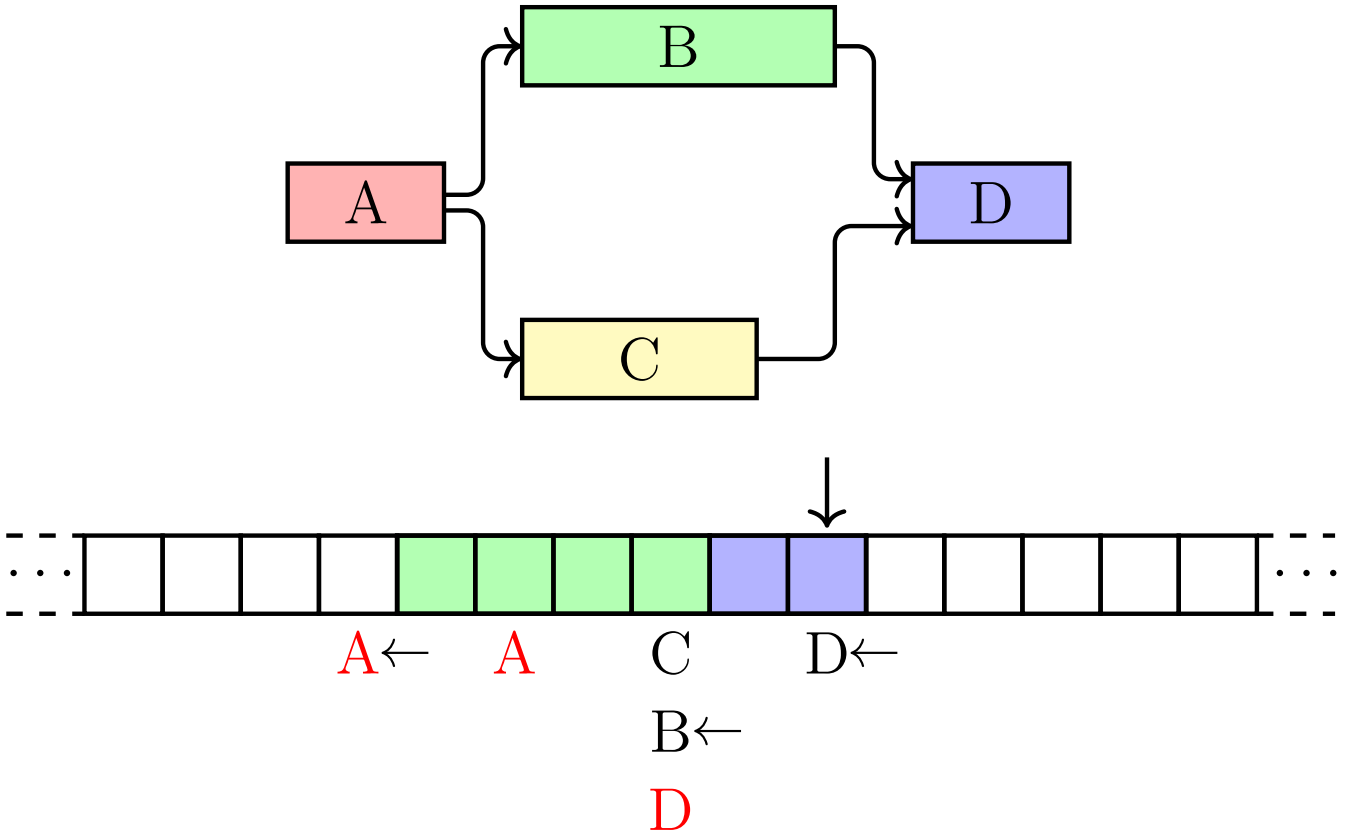

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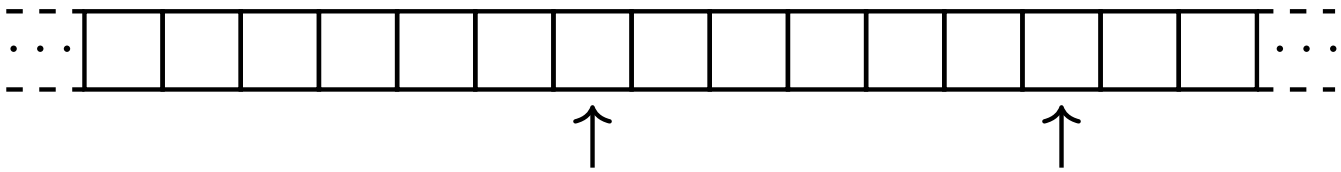

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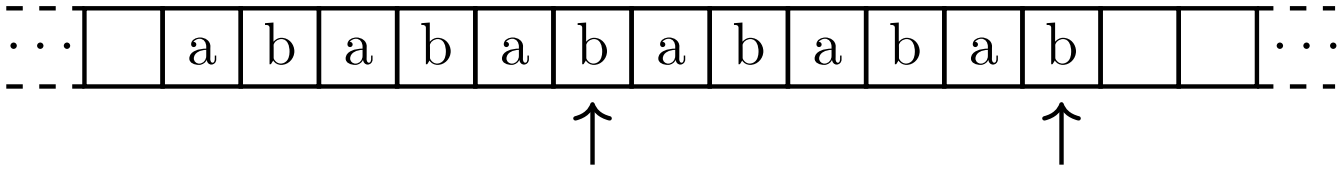

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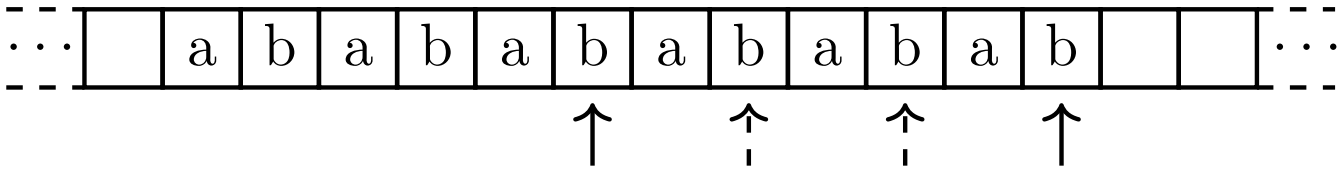

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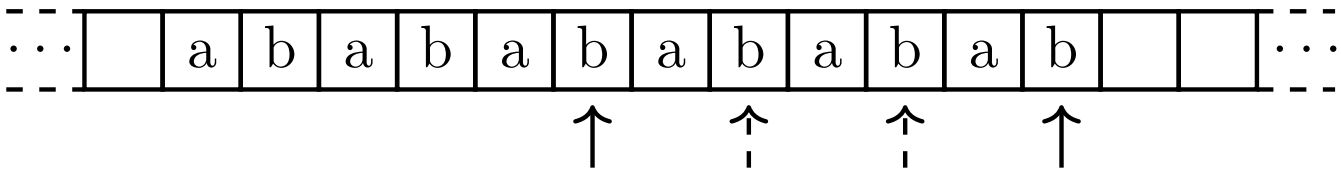

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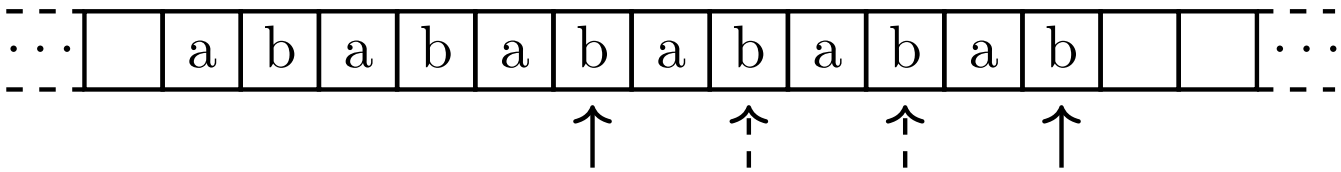

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In general, close occurrences of periodic strings happen in arithmetic progression, which can be stored efficiently.

Precomputing Backtracking Queries

Quick recap

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Solution

The content of a arithmetic progression is known beforehand, so queries can be precomputed!

Precomputing Backtracking Queries

Quick recap

- for the occurrences that are isolated, we store explicitly whether they're witnesses
- for the occurrences that happen in arithmetic progressions, we store the arithmetic progression and few witnesses at the beginning of the stream

But backtracking on long arithmetic progressions is (spacewise) expensive.

Solution

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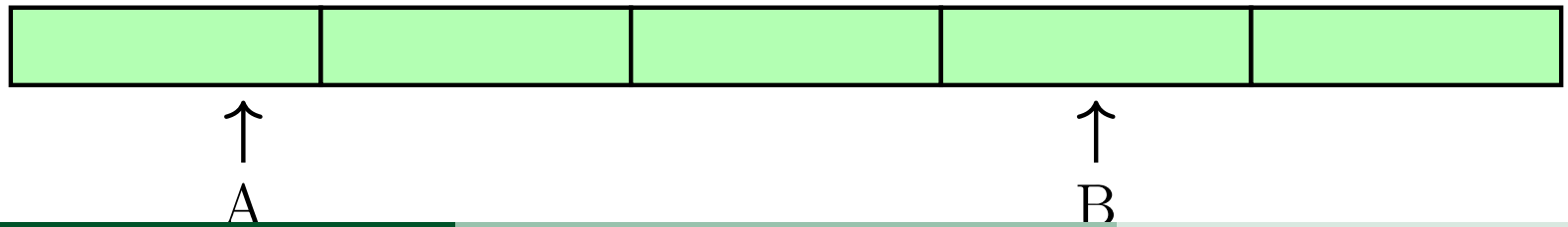

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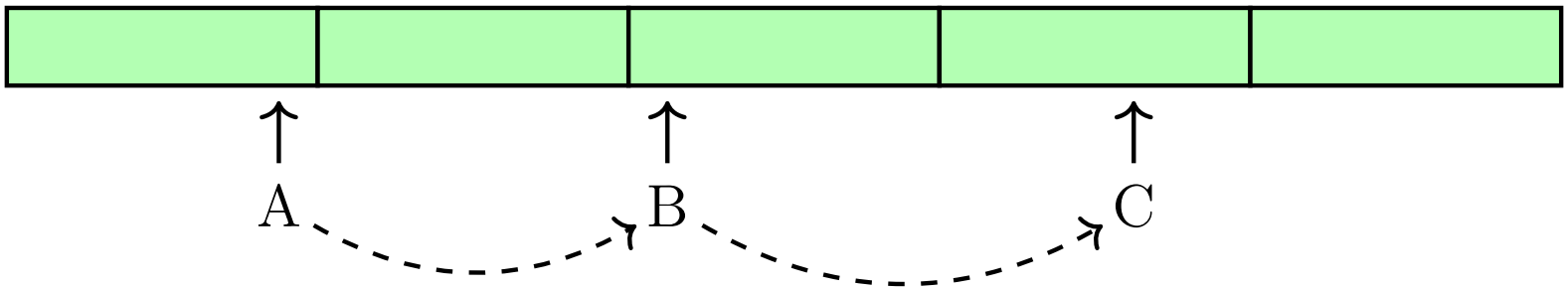
Each query can be translated in knowning whether there is a path of a given length between two nodes in this graph.



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We solve this problem by computing, for a given k, for all pairs of nodes (u, v), for all weights $w \leq 2^k$, whether there is a path from u to v of weight w.

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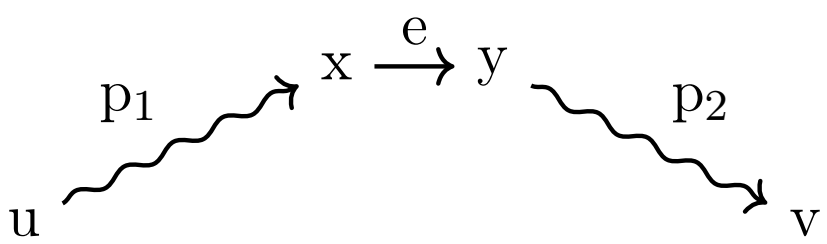

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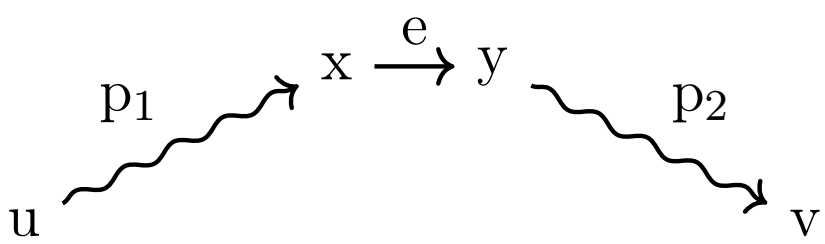
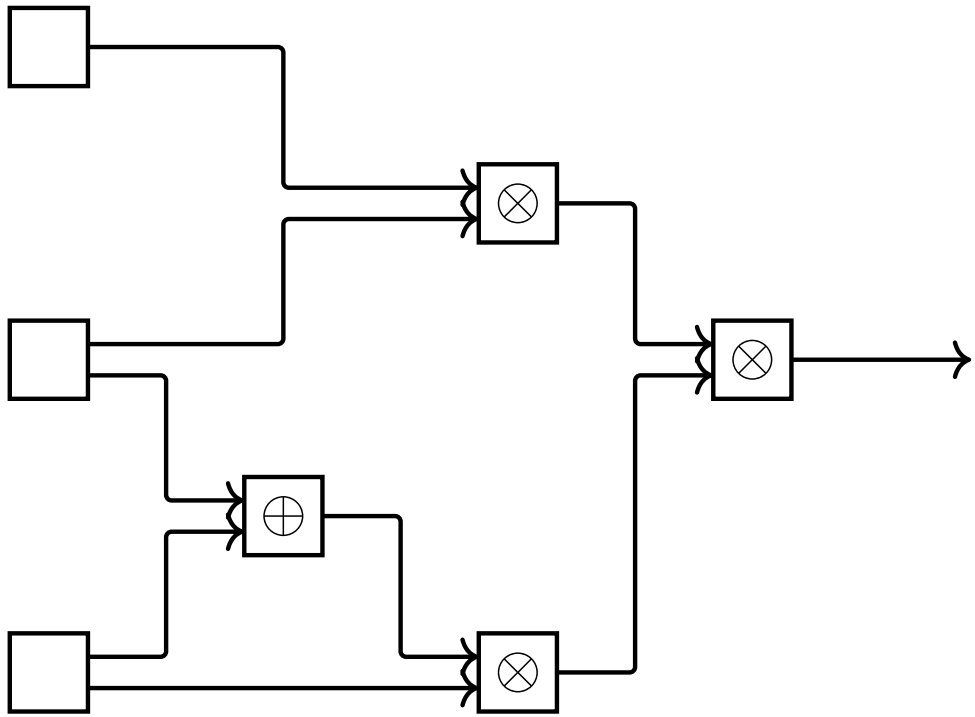


This amounts to iterated convolutions of solutions of the problem for a given k, to get the solutions for $k + 1$.

Reducing to a Circuit Problem

We translate the previous problem into a circuit with convolution and addition gates.



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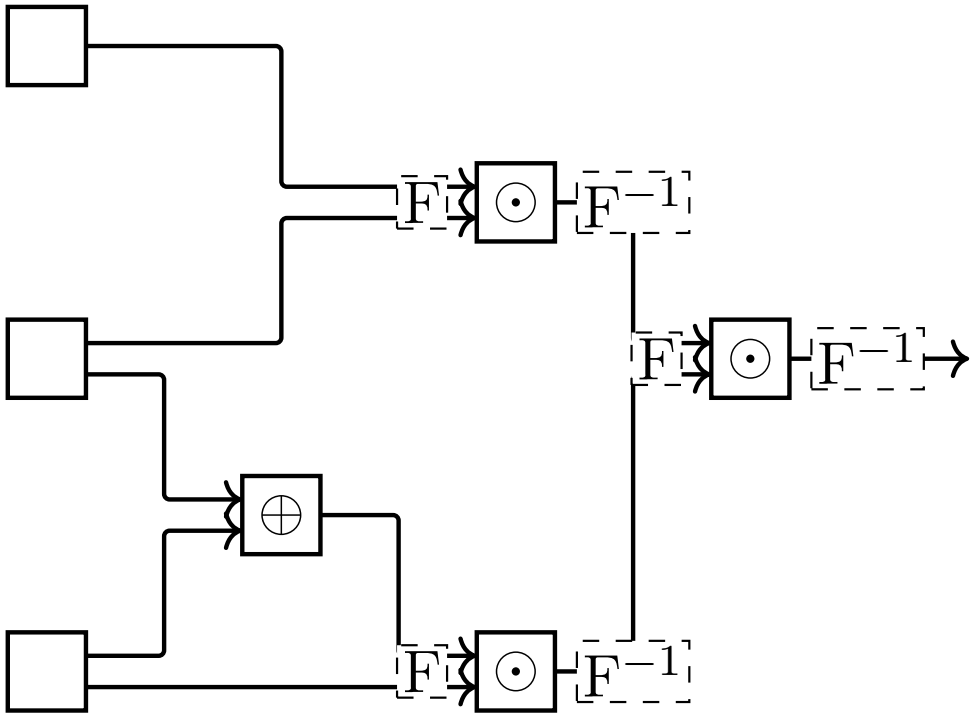

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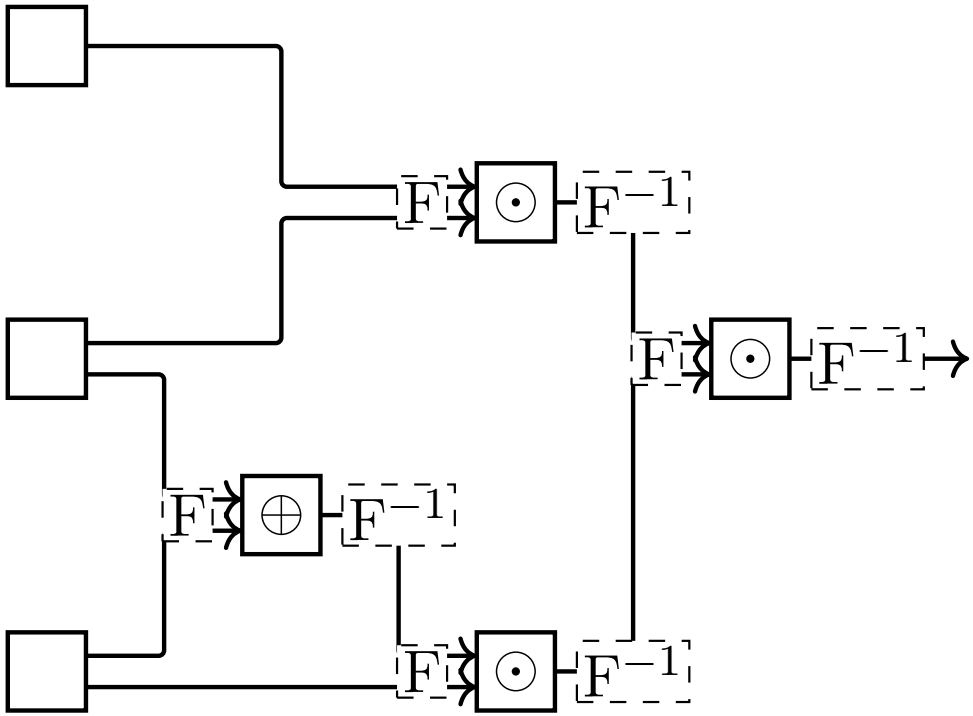

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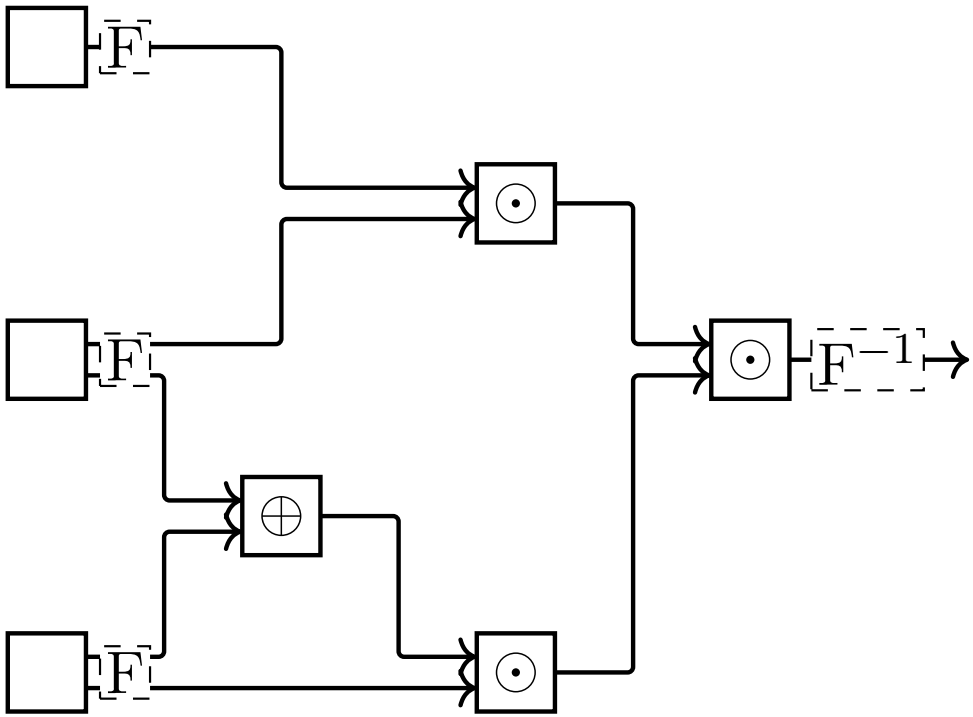

Where are we going?

1. Exact Regular Expression Matching
   - Solution Sketch
   - Witnesses
   - Space-Time Tradeoff
   - Backtracking Solution
   - Occurrence Structure
   - Precomputing Backtracking Queries
   - Reducing to a Graph Problem
   - Reducing to a Circuit Problem
2. Generalizing to Approximate Matching
   - General Idea
   - Handling Arithmetic Progressions
   - The Stubborn Edge Case

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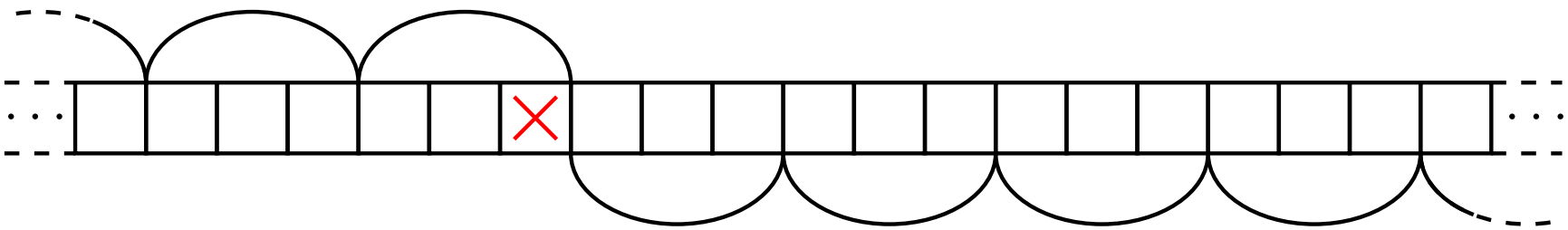
- we replace in the exact algorithm the exact string pattern matching routine with an approximate one;
- all the witnesses also store the number of mismatches;
- the graph-problem solving subroutine can also constrain the number of mismatches along the path;

Most of this works for any number of mismatches but, due to a single edge case, we only consider the case of one mismatch.

Handling Arithmetic Progressions

In the exact setting, the invariant is that all occurrences of an atomic string happen in an arithmetic progression streak. This is not true anymore:



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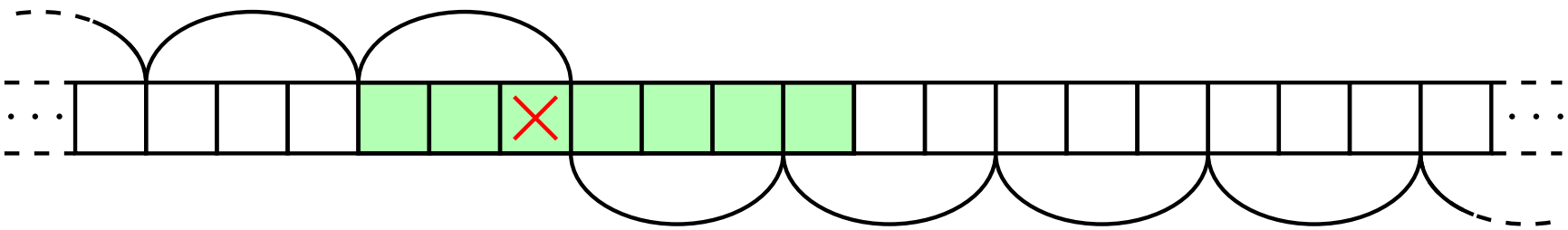

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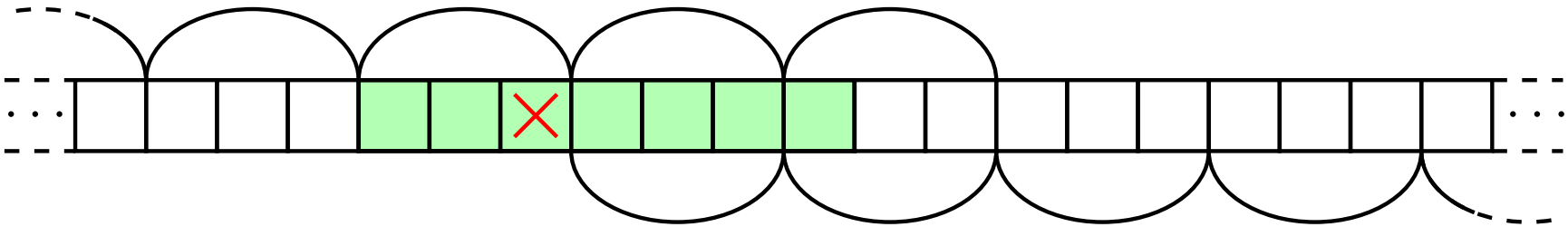
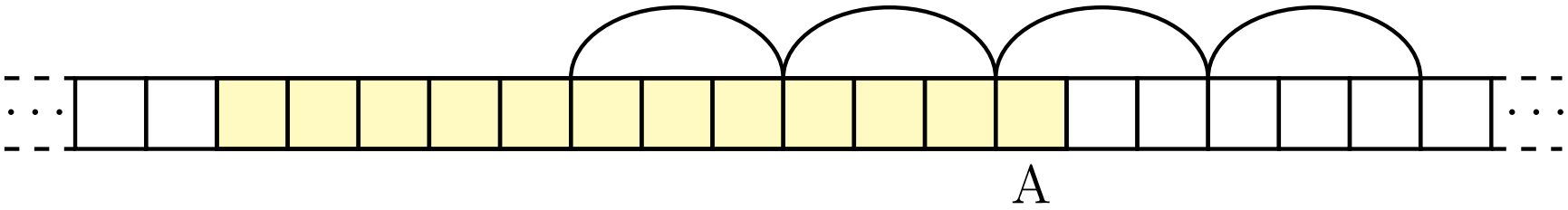


The Stubborn Edge Case

The annoying edge case is that of a long atomic string A that overlaps with an arithmetic progression.



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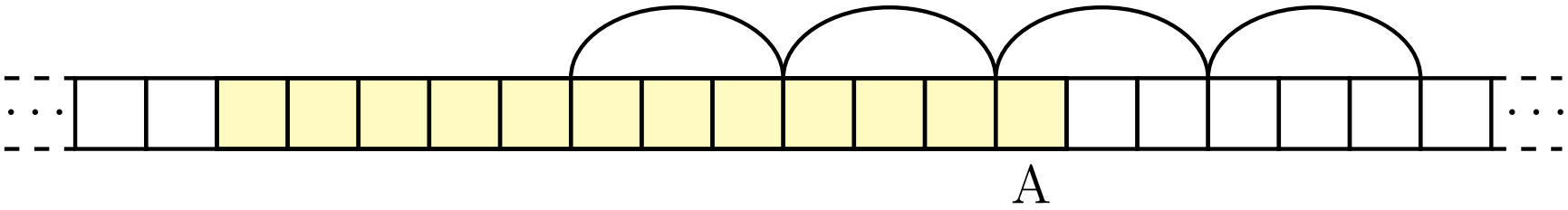

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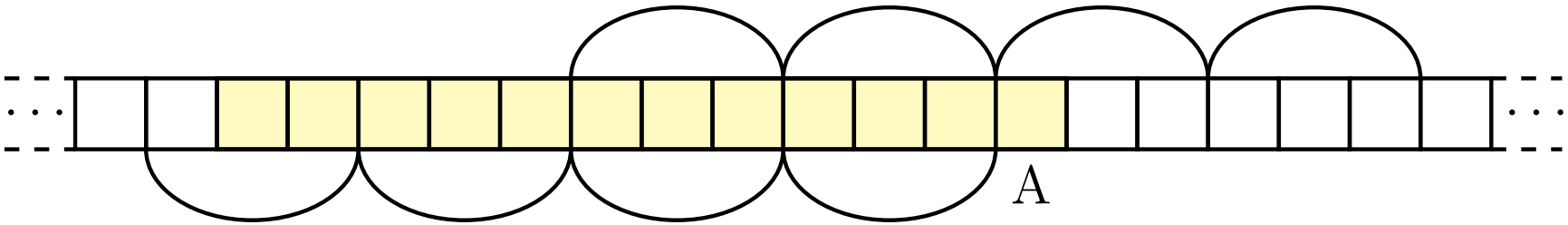

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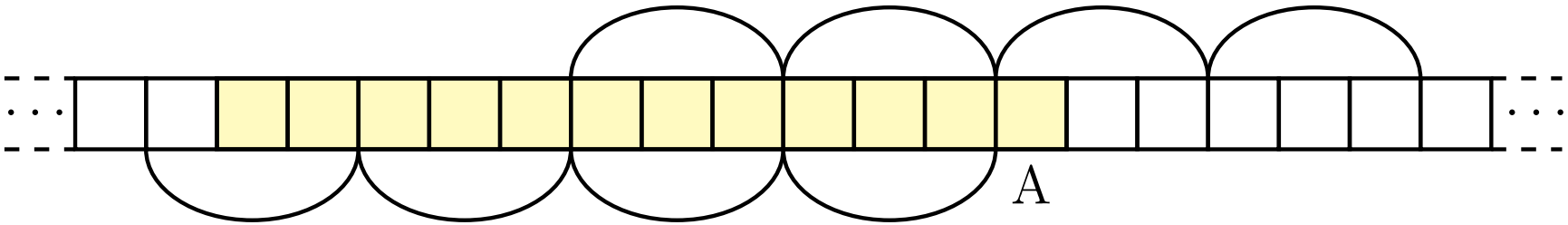

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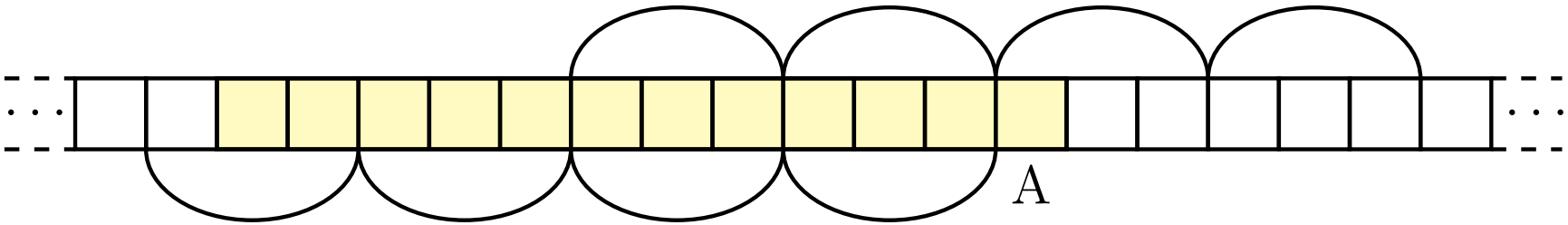

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Solution

- we store the arithmetic progression of occurrences of the first half of A that are close to the beginning of the overlap;
- we "guess" an occurrence of A;
- we check whether it's a witness.